Basics of spin wave theory

Sylvain Petit[1]

[1] LLB, CEA-Saclay, F-91191 Gif sur Yvette Cedex, France

Abstract*. This lecture is an introduction to the spin wave theory. We study the cases of a ferromagnet and of an antiferromagnet. We also discuss the use of neutron techniques in measuring the dispersion of spin waves.*

Introduction

Developed in the 50's, the spin waves theory is one of the milestones in magnetism and continues to be of fundamental importance. This approach allows a physically appealing treatment of long range magnetically ordered phases, where the quantum effects enter as time dependent fluctuations about the classical ground state. Spin waves can thus be seen as time dependent phenomena, actually corresponding to precession modes of the magnetically ordered structure, with typical energies of a few meV (or THz).

Basic assumptions

The theory is based on the Heisenberg Hamiltonian:

$$H = \sum_{m,i,n,j} S_{m,i}\, J_{m,i,n,j}\, S_{n,j} + \sum_{m,i} D_i \left(n_i\, S_{m,i}\right)^2$$

and takes into account exchange couplings $J_{m,i,n,j}$ between spins S_{mi} located at sites (m,i) and (n,j). Here, i is an index over the spin position within the unit cell (denoted by the index m). The second term is a single ion anisotropy term. As we want to minimize the energy, it is clear that if D_i is negative, n_i is a local easy-axis direction (at site i): the spins prefer to align along this direction. In contrast, if D_i is positive, n_i is perpendicular to an easy-plane, and the system gains energy if the spins lie down into that plane.

The basic assumption of the spin wave theory is that we can select a classical ground state and determine fluctuations around it. The spin waves dispersions can thus be obtained from different methods, the simplest being probably the linearized classical equations of motion. We shall however rederive these dispersions using Holstein-Primakoff bosons in a practical. Spin waves are thus independent harmonic oscillators which can be quantized in a usual way.

In the case of a ferromagnet, the long wavelength limit of the dispersion is

$$\omega_k = \mathrm{z}\,\mathrm{J}\,\mathrm{S}\,(1-\gamma_\mathrm{k})$$

with $\gamma_k = \frac{1}{\mathrm{z}}\sum_\mathrm{z} \mathrm{e}^{\mathrm{i}\,\mathrm{k}\,\Delta_\mathrm{z}}$ and z in the number of nearest neighbors. As k tends to zero, we find a gapless Goldtsone mode, which is a consequence of the spontaneous broken symmetry of the ferromagnetic ground state. We also calculate the temperature correction to the magnetization. The most important conclusion emerging from the theory is that, because of quantum fluctuations, these corrections diverge in 1 and 2 dimensions. As a result, our basic assumption of small fluctuations around a classical ordered ground state is wrong in those cases. This breakdown is consistent with the Mermin and Wagner theorem which states the absence of long range ordered phases in 1 and 2 dimensions. In 3 dimensions, the theory predicts a $T^{3/2}$ temperature correction to the low temperature magnetization.

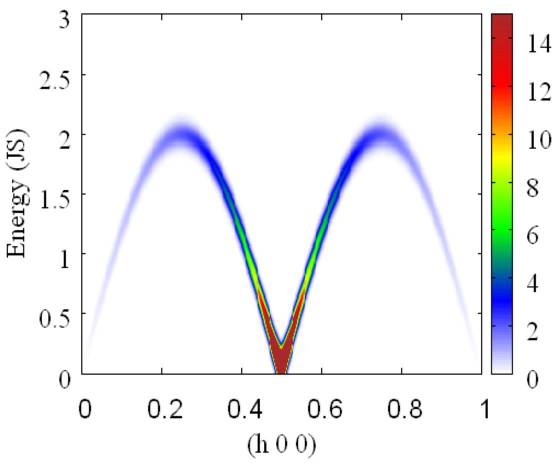
In the antiferromagnetic case, we have to define two sublattices, and this leads to a somehow more complicated theory. The main difference with the ferromagnetic case is that we now have a quantum zero point term in the energy. These corrections reduce the energy of the antiferromagnet compared to the Néel energy. The dispersion reads as

$$\omega_k = z\, J\, S \sqrt{(1-\gamma_k^2)}$$

and we find two Goldstone mode, one around k=0, and one at the Brillouin zone boundary (k=π). The (staggered) magnetization diverges in 1 dimension, signaling the failure of the theory and the absence of long range order in the ground state.

Measuring spin waves with neutrons

The spin wave dispersion is routinely measured by neutron spectroscopy, providing information about the coupling between spins and about anisotropy constants. Indeed, the neutron scattering cross section is proportional to the spin-spin correlation function which can be easily calculated within the theory. Based on numerical calculations, we shall discuss here several examples.



Practicals

Two practicals are proposed during the school. The first one is devoted to the Holstein-Primakoff approximation; the second one proposes numerical calculations of spin waves in a variety of systems, like for instance triangular lattice. We shall also study the folding of dispersions in a super-lattice.

References

[1] P.W. Anderson, Phys. Rev. 83, 1260 (1951).
[2] R. Kubo, Phys. Rev. 87, 568 (1952).
[3] T. Oguchi, Phys. Rev 117, 117 (1960).
[4] D.C. Mattis, *Theory of Magnetism I*, Springer Verlag, 1988.

[5] R.M. White, *Quantum Theory of Magnetism*, Springer Verlag, 1987.
[6] A. Auerbach, *Interacting electrons and Quantum Magnetism*, Springer Verlag, 1994.