Electron-phonon coupling: a tutorial

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This tutorial will start with an introduction of the harmonic oscillator and its solutions techniques in real space and second quantization.

The real space solution starts from the splitting of the asymptotes. The resulting real space Schrödinger equation is solved by the standard method of Taylor-series expansion which is terminated in order to comply with the asymptotes. The termination leads to a recursion relation that, in turn, quantizes the energy. The pertinent wavefunctions are Hermite polynomials. In energy space creation and annihilation operators of bosonic quanta are introduced which, by their ladder-operator properties, yield the same energy quantization as before. However, they give the additional feature of number counting (second quantization).

As a first step into electron-phonon coupling the harmonic oscillator is placed in an electric field and subsequently the modified solution of this problem is discussed. The result of the polaron problem obtained in this way is a shift of the equilibrium distance and the associated energy of the harmonic oscillator induced by the electric field. Generalizations of the electron-phonon coupling are the deformation potential and the piezoelectric coupling, which, along with the appropriate electron-phonon coupling matrix elements, will be discussed in some detail.

Next we address the Fröhlich Hamiltonian which has important implications for basic electron-phonon coupling as well as for the Bardeen-Cooper-Schrieffer theory of superconductivity. In addition we will discuss by means of small-polaron theory the case of small and large polarons. Finally we will conclude the subject by addressing the topic of phonons in metals, where electron-electron correlations along with electron-phonon interaction play an important role for an appropriate diagrammatic treatment of the associated effects.

Literature

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