Question:

How can I calculate the magnetostatic field from non-spherical particles such as cubes or octahedron? Furthermore, what happens if their magnetisation is not homogeneous?

Magnetostatic problem formulation

Maxwell's equations:

$$\vec{\nabla} \times \vec{H} = 0$$

$$\vec{\nabla} \cdot \vec{B} = \mu_0 \vec{\nabla}(\vec{H} + \vec{M}) = 0$$

$$\vec{H} = -grad\,\phi_{magn}$$

$$\vec{\nabla} \times \vec{H} = 0$$

$$\vec{\nabla} \cdot \vec{H} = \rho_m = -div\vec{M}$$ Volume charges

$$\Delta\phi_{magn} = \begin{cases} \nabla\vec{M} & inside \\ 0 & outside \end{cases}$$

Boundary conditions:

$$\phi_{magn}^{outside}\Big|_{\Sigma} = \phi_{magn}^{inside}\Big|_{\Sigma}$$

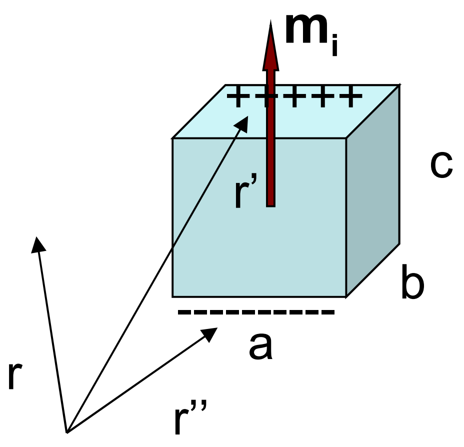
$$\left(\frac{\partial \phi_{magn}^{inside}}{\partial \vec{n}} - \frac{\partial \phi_{magn}^{outside}}{\partial \vec{n}} \right)_{\Sigma} = \sigma_m = \vec{M}\vec{n}$$ Surface charges

$$\phi(\mathbf{r}) = \frac{1}{4\pi}\int_V \frac{\rho_m(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|} d^3r' + \frac{1}{4\pi}\oiint_S \frac{\sigma_m(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|} ds'$$

Coulomb's law

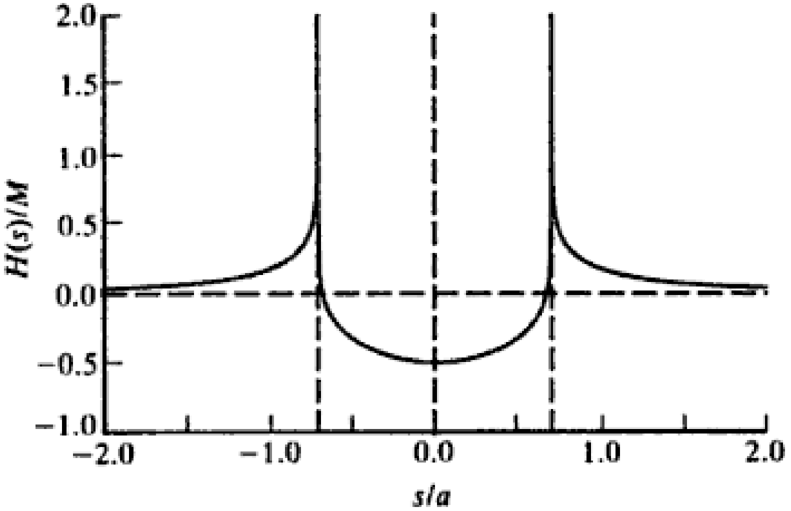
$$\mathbf{H_D}(\mathbf{r}) = \frac{1}{4\pi}\int_V \frac{(\mathbf{r}-\mathbf{r}')\rho_m(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|^3} d^3r' + \frac{1}{4\pi}\oiint_S \frac{(\mathbf{r}-\mathbf{r}')\sigma_m(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|^3} ds'$$

Saturated cube



$$\phi(\vec{r}) \propto \frac{M}{4\pi}\left(\iint_{\Sigma} \frac{dx'dy'}{\left|\vec{r}-\vec{r}'\right|} + \iint_{\Sigma'} \frac{dx''dy''}{\left|\vec{r}-\vec{r}''\right|}\right)$$

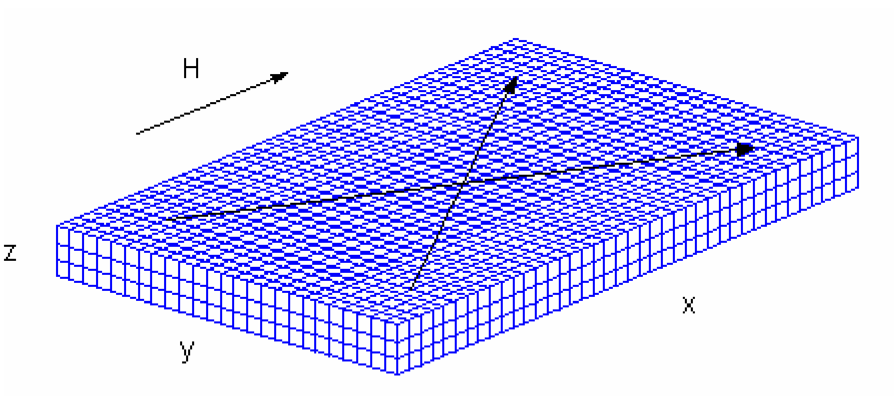
$$\vec{H} = -grad\,\phi$$

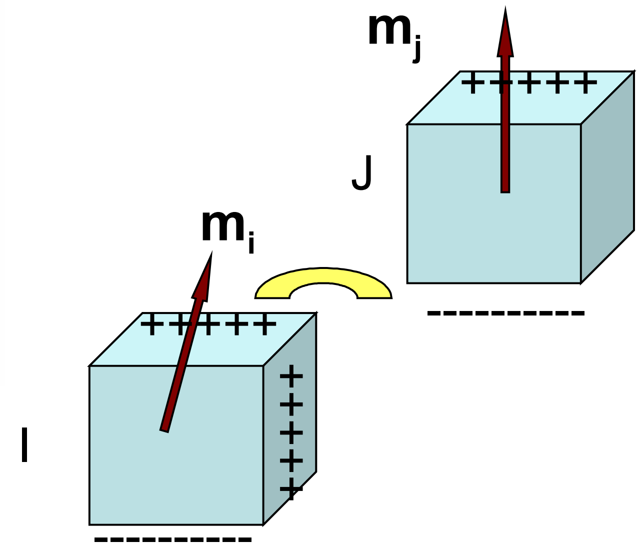


•The magnetostatic fields are stronger at the corners.

•This expression will not be valid is the magnetisation process is not homogeneous

Non-homogeneous processes: numerical micromagnetism



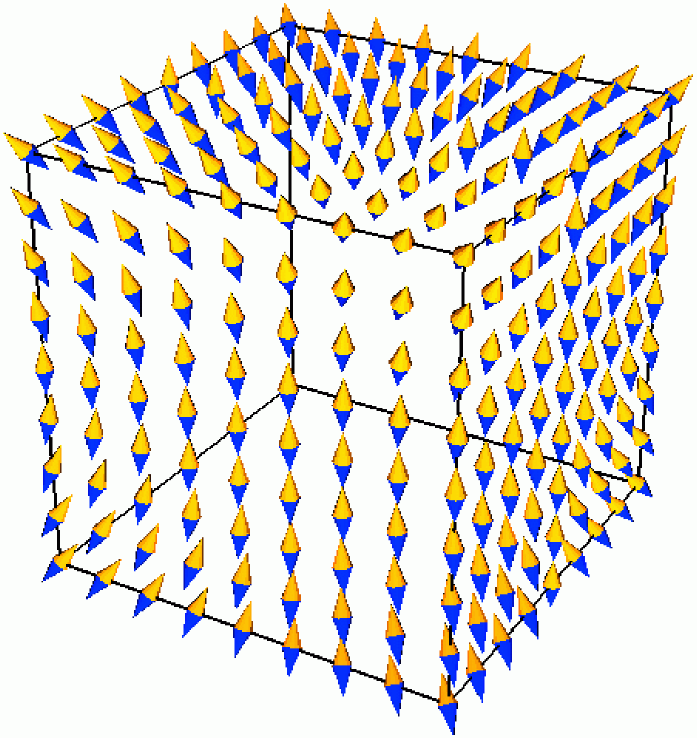


- Constant magnetisation in each discretization element
- Sum of all surface charges

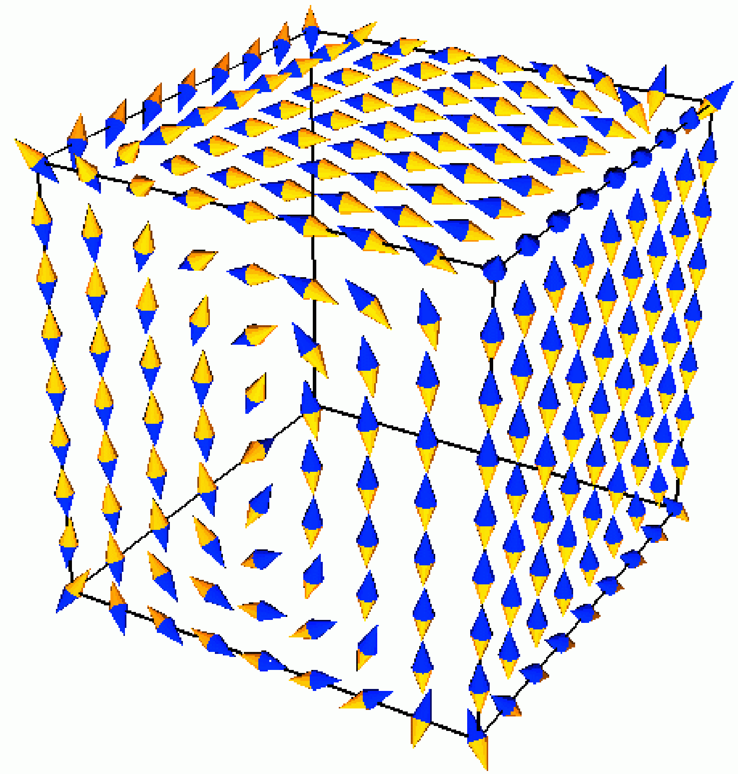
$$\vec{h}_i = \sum_j B_{ij}(\vec{r}_i - \vec{r}_j)\vec{m}_j$$

Precomputed matrix

μ-mag standard problem (from oommf webpage)



Longitudinal flower state



Transverse vortex state

See also M.A.Schabes abd H.N.Bertram “Magnetisation processes in ferromagnetic cubes”, J.Appl. Phys. 64 (1988) 1347.