

Magnetism

- 1 Paramagnetism
- 2 Magnetic order

Introduction ● magnetism is nothing new.....



History of Magnetism and Electricity

600 BC - Lodestone

The magnetic properties of natural ferric ferrite (Fe_3O_4) stones (lodestones) were described by Greek philosophers.

600 BC - Electric Charge

Amber is a yellowish, translucent mineral. As early as 600 BC the Greek philosopher, Anaximander was aware of its peculiar property: when rubbed with a piece of fur, amber develops the ability to attract small pieces of material such as feathers. For centuries this strange, inexplicable property was thought to be unique to amber. This strange effect remained a mystery for over 2000 years, until, around AD 1600, Dr William Gilbert investigated the reactions of amber and magnets and first recorded the word "Electric" in a report on the theory of magnetism.

Later in, in 1895, H.A. Lorentz developed the Electron Theory. We now know that there are three ways to generate electricity: Static, Electrochemical and Electromagnetic Induction.

1175 - First Reference to a Compass

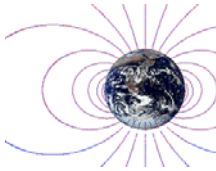
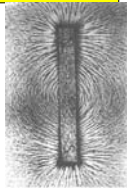
Alexander Necklem an English monk of St. Albans describes the workings of a compass.

Our first contact with magnetism...

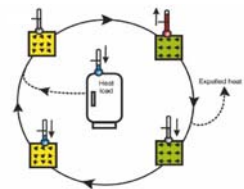


Join the engine to the three cars. Roll off again and combine the cars and a padded fabric house to

1 1/2+
15 pieces



Technology



Magnetic Resonance Imaging



Plasma spectroscopy, Zeeman effect



Basic properties 1

Quizzzzz

- What are the three sources of atomic magnetism and sort them by size
?
- What is Lenz's law?
?
- How large is the magnetic field you can generate with:
a) A refrigerator magnet
b) An iron horseshoe magnet
c) A superconducting magnet
- Which material reacts strongest on an applied magnetic field?
a) Argon
b) Na
c) O₂
d) Fe₃O₄

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Basic properties 1

Ferromagnetism of the elements

Room-temperature magnets

All other elements are:
Paramagnetic
or
Diamagnetic

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Basic properties 1

5 different types of magnetic materials

Temperature dependence of magnetic susceptibility $\chi_m(T)$

$$\mathbf{M}(H, T) = \chi_m(T)\mathbf{H}$$

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Basic properties 1

Names and definitions

H: magnetic field / magnetic field strength
B: magnetic induction / magnetic field

In vacuum $\mathbf{B} = \mu_0\mathbf{H}$

Material in magnetic field \mathbf{H} $\mathbf{B} = \mu_0(\mathbf{H} + \mathbf{M})$

Magnetization $\mathbf{M} = \frac{\mu_{tot}}{V}$

Magnetic susceptibility $\mathbf{M} = \chi_m\mathbf{H}$

K_m = relative permeability $\mu = K_m\mu_0$ $\chi_m = K_m - 1$

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Basic properties 1

Relative permeability @ 20°C

Material	$\chi_m = K_m - 1$ (x 10 ⁻⁵)	Material	$\chi_m = K_m - 1$ (x 10 ⁻⁵)
Paramagnetic			
Iron oxide (FeO)	720	Diamagnetic	
Iron ammonium alum	66	Ammonia	-26
Uranium	40	Bismuth	-16.6
Platinum	26	Mercury	-2.9
Tungsten	6.8	Silver	-2.6
Cesium	5.1	Carbon (diamond)	-2.1
Aluminum	2.2	Carbon (graphite)	-1.6
Lithium	1.4	Lead	-1.8
Magnesium	1.2	Sodium chloride	-1.4
Sodium	0.72	Copper	-1.0
Oxygen gas	0.19	Water	-0.91

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Basic properties 1

Paramagnetic susceptibility of localized electrons

$$\mathbf{M} = \chi(T)\mathbf{H}$$

$\chi > 0$
M parallel to H
→ Attractive force

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Basic properties 1

localized electrons

Some valence shells lie deep in the atom
 → have little overlap with neighbor atoms
 → atomic orbital moment "survives" band formation

Which electrons of the elements are these:

- 3d transition metals
- 4f Rare-earth metals
- 5f Early actinides

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Basic properties 1

Champion in localization: RE 4f shell

Rare earth:
 Electron configuration $4f^N 5d^1 6s^2$
 Thus three open valence shells!

Radial expectation value of hydrogen orbitals

little overlap with neighbor atom

<http://winter.group.shef.ac.uk/orbitron/AOs/4f/e-density-xzz-dots.html>

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Basic properties 1

Magnetic dipole moment of atom with 1 electron:

Electron has spin and orbital moment contributing each to the magnetic dipole moment

$\mathbf{l} = \mathbf{r} \times \mathbf{p}$

$\boldsymbol{\mu}_{orbit} = -\frac{e}{2m} \mathbf{r} \times \mathbf{p} = -\mu_B \mathbf{l}$

$\boldsymbol{\mu}_{spin} = -\mu_B g_0 \mathbf{s}$

$\mu_B = \frac{e\hbar}{2m} = 9.2 \times 10^{-24} \text{ J/T}$

$g_0 = 2.0023$

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Basic properties 1

Orbital moment in field // z axis

$\boldsymbol{\mu}_l = -\mu_B \mathbf{l}$
 $\mu_{l,z} = -\mu_B m_l$

$l = 0 (s)$
 $l = 1 (p)$
 $l = 2 (d)$
 $l = 3 (f)$

$m_l = 0, 1, 2, 3$

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Basic properties 1

Orbital moment in magnetic field

Zeeman energy $-\boldsymbol{\mu}_l \cdot \mathbf{B} = \mu_B B m_l$

Zeeman splitting

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Basic properties 1

Spin moment in field // z axis

$\boldsymbol{\mu}_s = -g_0 \mu_B \mathbf{s}$
 $\mu_{s,z} = -\mu_B g_0 m_s$

$g_0 = 2.0023$

$\hbar S_z \cong \pm \frac{1}{2} \hbar$

Zeeman energy $-\boldsymbol{\mu}_s \cdot \mathbf{B} = \mu_B g_0 B m_s$

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Basic properties 1

Spin-orbit interaction

Nuclear reference frame

Electron reference frame:
rotating proton produces magnetic field
(Relativistic effect)

Spin-orbit energy

$$E_{spin-orbit} = \lambda \mathbf{l} \cdot \mathbf{s}$$

$$\lambda \sim \frac{Z^2}{n(l+1)(l+\frac{1}{2})}$$

for state with quantum number n, l :

- No splitting if $l=0$ (s orbitals)
- Increases with Z , decreases with n, l
 - 10-100 meV for valence shell
 - 10-1000 eV for inner shell

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Basic properties 1

Spin-orbit splitting

- Spin orbit interaction leads to splitting of levels
- New eigenstates with quantum number $j=l+s$ or $j=l-s$
 $m_j = j, j-1, \dots, -j$

Labeling: l_j

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Basic properties 1

Total moment of valence shell n^N

The total magnetic moment is the sum of all magnetic dipole moments

$$\boldsymbol{\mu} = -\mu_B (\mathbf{L} + 2\mathbf{S})$$

$$\hbar \mathbf{L} = \hbar \sum_{i=1}^N \mathbf{l}_i \quad \text{Total atomic orbital-moment}$$

$$\hbar \mathbf{S} = \hbar \sum_{i=1}^N \mathbf{s}_i \quad \text{Total atomic spin-moment}$$

Full shells have no dipole moment: $ns^2, np^6, nd^{10}, nf^{14}$
Partly filled shells have permanent dipole moment

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Basic properties 1

Multiplet splitting of np^2 configuration

Configuration	+Coulomb interaction	+Spin-orbit interaction	+magnetic field (Zeeman splitting)
np^2	1S	1S_0	<ul style="list-style-type: none"> Multiplet state Hund's rule ground state (here $J=0$)
		1D	
	3P	3P_2	

Energy scale: ~ 10 eV (Coulomb), $0-1$ eV (Spin-orbit)

Spectroscopic notation: $2S+1L_J$

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Basic properties 1

Coupling schemes for adding orbital and spin angular momenta

- Russel Saunders or L-S coupling** if spin-orbit interaction is weak \rightarrow eigenstates of the atom are also eigenstates of L^2 and S^2 with eigenvalues $L(L+1)$ and $S(S+1)$.
 - Add up all orbital moments $\mathbf{L} = \sum_{i=1}^N \mathbf{l}_i$
 - Add up all spin moments $\mathbf{S} = \sum_{i=1}^N \mathbf{s}_i$
 - Add these together to total atomic angular moment $\mathbf{J} = \mathbf{L} + \mathbf{S}$

3 schemes for coupling of moments

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Basic properties 1

Coupling schemes for adding orbital and spin angular momenta

- j-j coupling** if spin-orbit interaction is dominant
 - Add up orbital and spin moments of each electron $\mathbf{J}_i = \mathbf{l}_i + \mathbf{s}_i$
 - Add these \mathbf{J}_i together to $\mathbf{J} = \sum_{i=1}^N \mathbf{J}_i$
- Intermediate coupling** complicated no fixed rule

3 schemes for coupling of moments

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Basic properties 1

Hund's rules for J of ground state

Procedure for J of the ground state (L-S coupling):

1. Maximize $S = \sum s_{z,i}$
2. Maximize $L = \sum l_{z,i}$
3. $J = |L - S|$ for less than half filled shell
 $L + S$ for more than half filled shell
 if subshell is exactly half-filled, $L = 0$, so $J = S$

example:
 3d transition metals:
 Fill the 3d orbital (l=2)

The graph shows the values of S, L, and J for 3d orbitals as a function of the number of electrons (0 to 10). S (blue line) increases from 0 to 5. L (red line) increases from 0 to 5, then decreases to 3, then increases to 4, then decreases to 2, then increases to 3, then decreases to 1, then increases to 2, then decreases to 0. J (yellow line) follows the same pattern as L.

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Basic properties 1

Zeeman effect for state with total moment J

- Ground state J is $2J+1$ times degenerated: $J_z = -J, -J+1, \dots, J$
- Splits in magnetic field into sublevels

The diagram shows a single energy level J splitting into $2J+1$ sublevels with $J_z = 2, 1, 0, -1, -2$. The energy difference between adjacent sublevels is $\Delta E = g_L \mu_B B_z$.

$$H_p = -\boldsymbol{\mu} \cdot \mathbf{B} = -\mu_z \cdot B$$

$$E = \langle H_p \rangle = g_{Lande} \mu_B J_z B_z$$

$$g_{Lande} = \frac{3}{2} - \frac{L(L+1) - S(S+1)}{2J(J+1)}$$

- Spectroscopic splitting factor g_{Lande} depends on L, S, and J
- Splitting at $B=1$ Tesla in the order of meV
- Atom behaves as if it has effective moment: $\mu_{eff} = -g_L \mu_B \mathbf{J}$

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Basic properties 1

Effective moment often called m_J

- once we have, \mathbf{J} , we can get the maximum component of the magnetic moment of the atom parallel to the magnetic field:

$$m_J = -g \mu_B J$$

g , Landé splitting factor:

$$g = \frac{3J(J+1) + S(S+1) - L(L+1)}{2J(J+1)}$$

pure orbital motion = 1, pure spin = 2

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Basic properties 1

Multiplet + magnetic field: Zeeman effect

Configuration	+Coulomb interaction	+Spin-orbit interaction	+magnetic field (Zeeman splitting)
np^2	1D	1D_2	Splitting typically 10 meV
	3P	$^3P_2, ^3P_1, ^3P_0$	Typically 1 eV

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Basic properties 1

Temperature dependence

The graph shows the magnetic moment $M/N\mu_B$ as a function of B/T (T/K) for Gd^{3+} , Fe^{3+} , and Cr^{3+} . The Gd^{3+} curve reaches a saturated moment of 7. The Fe^{3+} curve reaches a saturated moment of 5. The Cr^{3+} curve reaches a saturated moment of 3. The Brillouin function is shown for comparison.

magnetic moment of system with N ions at T is determined by Boltzmann statistics:

$$M = \frac{\sum_{J_z=-J}^J -g\mu_B J_z \exp(-g\mu_B B J_z / k_B T)}{\sum_{J_z=-J}^J \exp(-g\mu_B B J_z / k_B T)}$$

$$M(B, T) = N\mu_B g J B_J \left(\frac{g\mu_B B J}{k_B T} \right) \quad B_J \text{ Brillouin function}$$

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Basic properties 1

Curie's law

$$M(B, T) = N\mu_B g J B_J \left(\frac{g\mu_B B J}{k_B T} \right)$$

for not too small T:
 Curie's law

$$\chi_m(T) = \frac{Np^2 \mu_B^2 \mu_0}{3k_B T}$$

$p_J = g_{Lande} [J(J+1)]^{\frac{1}{2}}$
 Effective number of magnetons

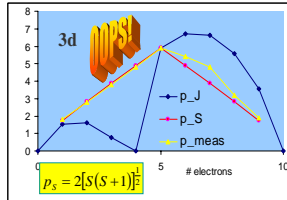
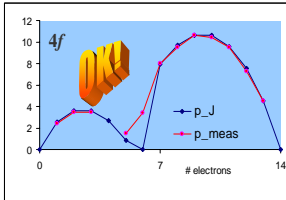
The graph shows the magnetic susceptibility $\chi_m(T)$ as a function of B/T (T/K) for Gd^{3+} , Fe^{3+} , and Cr^{3+} . The curves show a $1/T$ dependence at low temperatures.

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How well works Hund's rule?

Effective number magnetons

$$\rho_J = g_{Lande} [J(J+1)]^{\frac{1}{2}}$$



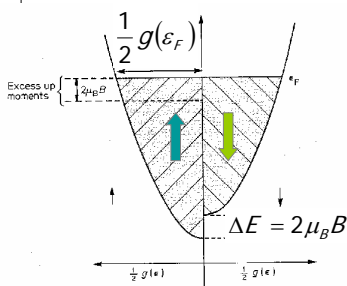
In 3d transition metals delocalization destroys orbital moment (quenching)

Paramagnetism of band electrons

- Up to now: localized electrons
- When valence electrons form a band, orbital moment is quenched
- Yet we see a paramagnetic moment in metals
- Explained by Wolfgang Pauli



Paramagnetism of free electron gas



Field lifts spin degeneracy

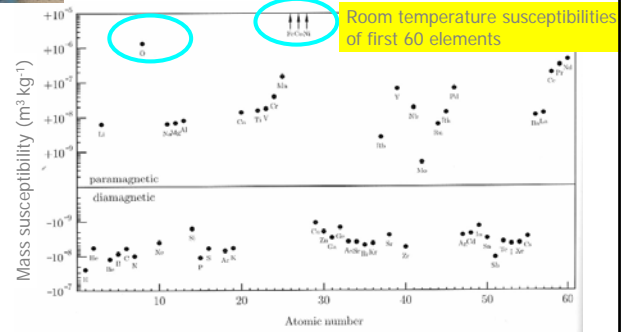
Estimate:

$$M \approx \mu_B \Delta n = \mu_B^2 g(\epsilon_F) B$$

$$\chi(T) = \frac{M}{H} = \mu_0 \mu_B^2 g(\epsilon_F)$$

Pauli paramagnetism
Temperature independent

Susceptibilities are small



Typical density: 0.2 mol/cm³ → Molar susc. $\chi_{mol} = 10^{-4} \ll 1$