Scattering theory of transport in magnetic systems.

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In this lecture, I will study some interplays between the field of mesoscopic physics (which describes fully coherent electronic systems) and classical magnetism. I will start with a discussion of some classical mesoscopic experiments (Aharononv-Bohm effect, conductance quantification) which show non local behaviors. Those experiments helped to realize that the conductivity of a phase coherent system is a meaningless concept and only the conductance of the whole can be defined properly. Then I will introduce the Scattering theory of transport and the Landauer formula which relates the conductance to the scattering properties of the system. In the second part, the scattering approach will be applied to two different magnetic systems: a ferromagnet–normal-metal–ferromagnet trilayers and a magnetic domain wall. We will study the effect of magnetism on the transport properties as well as its counter part, the spin torque exerted on the magnetization by the conducting electrons.

I. AN ELEMENTARY INTRODUCTION TO TRANSPORT IN A MESOSCOPIC SYSTEM.

When do we really need quantum mechanics to describe the physical properties, let us say the conductance, of a solid state system? Take for instance a regular copper wire with a $1mm$ section and have a $1A$ current flowing through it. At what velocity do the copper's electrons go? The current density $j = 10^6 A.m^{-2}$, while the electronic density is $n_e \approx 10^{30}$ electron per m^3 (there is roughly one copper atom every Angstrom and each of them gives one valence electron.) Now if we use the classical relation $j = e\ n_e v$, we obtain that the average electronic velocity is $v \approx 10^{-5} m.s^{-1}$, which is two or three orders of magnitude too slow. What went wrong in that way of thinking is that we forgot completely the Pauli principle (and the Bloch theory of bands) which freezes the electrons far from the Fermi sea so that eventually no more than 1% of them are participating to the transport properties. In that sense, we need quantum mechanics to describe properly the transport properties of *any* electronic system although we know that our copper wire conductance is very well described by Ohm's law.

What one calls *mesoscopic* systems are systems where the electrons really behave both as particles and waves at the same time so that the predictions of quantum mechanics become strongly different from what one can obtain from semi-classical approaches, even at a qualitative level. In practice those "strongly quantum mechanical" devices are just regular electronic systems (such as the Field Effect Transistors that one can find in any PC) that are put in a dilution fridge at temperatures in the milli-kelvin range. At these temperatures many degrees of freedom of the system (like the phonons for instance) are frozen, and the phase of the wave function describing the electrons become a well defined quantity, hence the need for a fully quantum mechanical description.

A. Transport in a phase coherent system. Examples of "non local" experiments where quantum mechanics is fully in action.

In Fig.1, I have sketched two standard experiments that illustrate a few aspects of the conductance of a coherent object. In the upper one Fig.1 (a), one measures the conductance of a small metallic wire (a four points measurement: two contacts are used to inject the current I and the voltage difference V is measured on the other two.) connected to a small loop. When a magnetic field B is put through the system the conductance g shows oscillations. This experiment teaches us three things:

- (i) If this system was to be described by Ohm's law, the electrons would go directly from the $+I$ electrode to the $-I$ electrode and would never see the loop. Hence, the conductance of such a system would not depend on the magnetic field B. In other word, there is no way we can define a conductivity for this system, and we have to consider it as a whole.
- (ii) the oscillations die at higher temperature when the phase of the wave function is not well defined. Indeed they are due to interference between trajectories going clockwise and counter clockwise in the loop and such an effect needs phase coherence.

- (iii) the magnetic field inside the metallic part is actually zero (we imagine we have a very thin solenoid going through the loop) while the flux inside the loop is not. Hence the system is not sensible to the magnetic field itself but to the vector potential, another manifestation of quantum mechanics.

In the second experiment (Fig.1 (b)), one measures the conductance of a two-dimensional electron gas (in a GaAs heterostructure for instance) connected to two electrodes. In top of the gas, two (triangular) gates have been deposited and, when polarized with a gate voltage V_g, they deplete the gas underneath creating a constriction. Such a system is known as a QPC (Quantum Point Contact). Remarkably, one observes that g exhibit plateaux as a function of V_g. Those plateaux are quantized in unit of $\frac{2e^2}{h} \approx (10k\Omega)^{-1}$. Here again, the wave nature of the electrons shows up, and the conductance quantification can be understood if one considers that the gates act as wave guides allowing only a few modes too propagate.

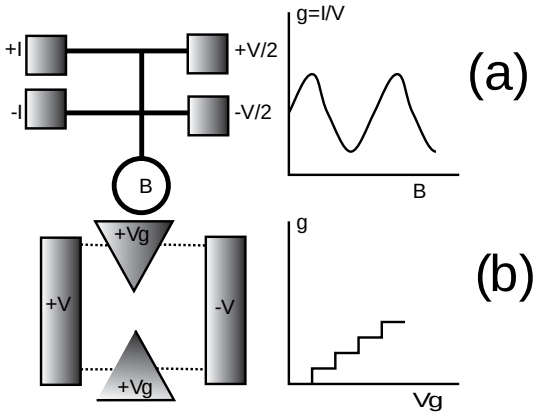

FIG. 1: Schematic of two experiments with mesoscopic samples. In (a) one measures the conductance g of a metallic wire connected to 4 contacts and one loop. g oscillates as a function of an Aharonov-Bohm flux put through the loop. (b) Quantum Point Contact (QPC): a two dimensional electron gas (inside the dotted lines) is connected to two electrodes at a potential $+V$ and $-V$. Two triangular gates are deposited in top of the gas and are used to deplete the electron underneath them and create a constriction. As a function of the gate voltage V_g used to polarize the gates, g exhibit steps.

At this stage, one should be convinced of the necessity of a quantum mechanical description of our system which raises a few theoretical questions. In particular, since the apparatus (and the experimentalist!) are classical objects, one needs to stop the quantum mechanical description at some point, and connect our quantum object to the classical world. The formalism that does that is the scattering theory of transport, and will be outlined in the next section.

B. The ballistic system as an "electronic wave guide". Introduction of the scattering matrix S. The Landauer formula for the conductance.

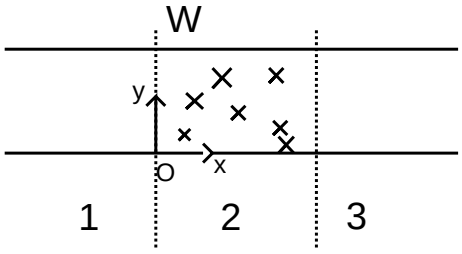

FIG. 2: In this ideal wire, the part 1 and 3 are ballistic while all the scattering takes place in region 2. The electrons are confined in the region $0 \leq y \leq W$ and the wire is attached to two ideal reservoirs.

We focus now on a two dimensional wire confined in the y direction on a width W as sketched in Fig.2. In part 1 and 3 of this system, the dynamics is ballistic (no potential) while all kind of imputities, roughness,...lie in region 2. The scattering theory of this system is done in two steps.

In step I, we want to study the solutions $\Psi(\vec{r})$ of its Schrödinger equation, hence doing plain quantum mechanics. The Schrödinger equation reads in this case,

$$-\frac{\hbar^2}{2m}\Delta\Psi(\vec{r}) + V(\vec{r})\Psi(\vec{r}) = E\Psi(\vec{r}) \tag{1}$$

where the potential $V(\vec{r})$ is zero outside region 2 and $\Psi(x, y = 0) = \Psi(x, y = W) = 0$ due to the confinement in the y direction. The solutions of Eq.(1) in region 1 and 3 are just a superposition of plane waves (we neglect the evanescent modes) with longitudinal and transverse momentum being respectively k_x and k_y. Due to the hard wall condition in the transverse direction, k_y is quantized in unit of $2\pi/W$ and can take $N_{\rm ch}$ (number of open channels) different values, that keep the mode propagating (i.e. $k_x = \sqrt{2mE/\hbar^2 - k_y^2}$ real).

$$x \in 1 : \Psi(x, y) = \sum_{n=1}^{N_{\rm ch}} \frac{\sin(k_y y)}{\sqrt{2W k_x}} \left[a_{1n} e^{ik_x x} + b_{1n} e^{-ik_x x}\right] \tag{2}$$

$$x \in 3 : \Psi(x, y) = \sum_{n=1}^{N_{\rm ch}} \frac{\sin(k_y y)}{\sqrt{2W k_x}} \left[a_{3n} e^{-ik_x x} + b_{3n} e^{ik_x x}\right] \tag{3}$$

Now, we don't know what $\Psi(x, y)$ looks like in region 2, but we do know that $\Psi(x, y)$ and its derivative are continuous at the $1-2$ and $2-3$ interfaces. Those conditions imply that a linear constraint links a_{1n} and b_{1n} to a_{3n} and b_{3n}. This constraint is written using the S matrix that relates the outgoing modes to the ingoing ones:

$$\begin{pmatrix} b_{1n} \\ b_{3n} \end{pmatrix} = S \begin{pmatrix} a_{1n} \\ a_{3n} \end{pmatrix} \quad \text{with} \quad S = \begin{pmatrix} r & t \\ t' & r' \end{pmatrix} \tag{4}$$

where S has been parameterized in term of the transmission (t,t') and reflection (r,r') submatrices. Probability current in region 1 (3) reads $\sum_n |a_{1n}|^2 - |b_{1n}|^2$ ($\sum_n |b_{3n}|^2 - |a_{3n}|^2$), hence current conservation enforces the orthogonality of the S matrix, $SS^\dagger = 1$.

Step II consists of giving ourself a prescription as to how those eigenstates are to be filled when the wire is attached to two reservoirs, each of them characterized by a temperature and a chemical potential. This prescription (which took quite a few years to emerge) is that the *incoming modes* coming from one reservoir have the equilibrium distribution given by this reservoir. With that prescription, we can now calculate physical quantities such as the current flowing through the wire. In the case of zero temperature and small bias voltage, we get for the conductance of the system,

$$g = \frac{2e^2}{h} \mathrm{Tr}\ tt^\dagger. \tag{5}$$

Eq.(5) is known as the Landauer formula. It relates the conductance of the system to its S matrix, and has been used widely in mesoscopic physics. Most of the remaining difficulties lie in trying to determine the properties of the S matrix. $tt^\dagger$ has $N_{\rm ch}$ eigenvalues $0 \leq T_n \leq 1$ called transmission probabilities, so that Eq.(5) can be rewritten as, $g = \frac{2e^2}{h} \sum_{n=1}^{N_{\rm ch}} T_n$.

C. Application to broken junctions. Experiments with paramagnetic and ferromagnetic metals.

As a first application of the Landauer formula, we can go back to the QPC described in Fig.1 (b). There the region 2 has no potential, and the transmission matrix t is just identity. The Landauer formula therefore reads $g = \frac{2e^2}{h} N_{\rm ch}$ and the conductance quantification appears naturally, the gate voltage controlling the number of opened channels. One should realize that this result is somehow paradoxal: the Landauer formula predicts an non zero resistance, *in the absence* of any scattering in the system. Not to mention that the quantum mechanical description made in the previous section is completely elastic, so that *a priori* the system has no way to dissipate the energy associated with the Joule effect.

The metallic equivalent of the QPC are broken junctions, where one inject current through a few metallic atoms. In contrast with the QPC, there the fermi wave length (roughly the "size" of a channel) coincide with the size of the atom. However, broken junctions give the possibility to study various metals, and magnetic ones in particular.

II. APPLICATION OF THE LANDAUER FORMULA TO MAGNETIC SYSTEMS

In this second part, we will apply the scattering theory introduced in the first one to magnetic systems. To do so, we need to make one modification to the previous theory, namely taking into account the spin structure of the S matrix,

$$S = \begin{pmatrix} S_{\uparrow\uparrow} & S_{\uparrow\downarrow} \\ S_{\downarrow\uparrow} & S_{\downarrow\downarrow} \end{pmatrix} \tag{6}$$

Otherwise, the formalism can be applied as it is. In addition to the charge current studied in the previous section, $I \propto \int dy \Psi(x,y) \partial_x \Psi(x,y)$ we will be interested in the spin current $\vec{J}$ flowing through the system, whose definition differs from the charge current by the presence of Pauli matrices: $\vec{J} \propto \int dy \Psi(x,y) \vec{\sigma} \partial_x \Psi(x,y)$.

A. Ferromagnetic-Normal Metal-Ferromagnetic trilayer. GMR. Spin injection and magnetization reversal.

Let us start with a system made of three metallic thin layers with a current flowing perpendicularly to the layers (see Fig.4). Two of those layers are magnetic while the spacer in between (that we will ignore in our theoretical description) is a normal metal. For the sake of simplicity, we suppose that the (majority) electrons whose spins are aligned with the local magnetization are fully transmitted while the (minority) electrons whose spins are antialigned are fully reflected.

With such a simple model, the conductance of the system is readily calculated. When the magnetization of the two layers are aligned, the majority spins are fully transmitted while the minority spins are reflected, hence $g = N_{\rm ch} e^2/h$. On the contrary, when the magnetizations are anti-aligned, the majority spins of one layer are the minority of the other and vice versa, hence $g = 0$. A magnetic field will bring the system from the anti-aligned to the aligned configuration, increasing the conductance. This phenomena is known as the Giant-Magneto-Resistance effect.

More interesting than the electronic current is what happens to the spin current in this system. Consider for instance a spin polarized current incident on a single magnetic layer such that the electrons' spins make an angle θ with the magnetization of the layer, as shown in Fig.3. Along the layer's magnetization, the incoming electrons' spins can be considered as a coherent superposition of majority spins with amplitude $\cos\theta/2$ and minority spins with amplitude $\sin\theta/2$. Hence, the electron will be transmitted with probability $\cos^2\theta/2$ and reflected with probability $\sin^2\theta/2$. The important point here, is that the incident spin current along the x-axis, proportional to $\sin\theta/2 \times \cos\theta/2 \propto \sin\theta$ is lost in the process. Hence, unlike the electrical current, *spin current is not conserved*. This is quite puzzling since we know that had we started our discussion with a full microscopic model, the total angular momentum of the system would have been bound to be conserved. Hence whatever spin current is lost by the conducting electrons has to be transferred to the electrons responsible for the magnetization, and a spin torque is exerted on the magnetization of the layer.

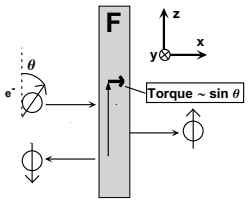

FIG. 3: Polarized electrons incident on a single magnetic layer.

In order to get this spin-torque in a real sample, you need polarized current. This is achieved by using not one but two magnetic layers, one thick one in charge of polarizing the current (hereafter labelled F_a), and a thin one upon which the torque is exerted (layer F_b). When the electrons enter the sample from the right, see Fig.4a, electrons transmitted from F_a will be polarized along its magnetization, hence exerting a torque on F_b. The torque, which is simply calculated as the difference of spin current on the two sides of F_b, tends to *align* the magnetization of F_b toward the one of F_a. The situation is quite different when the current is flowing in the other direction, see Fig.4b. There, the incident electrons coming on the left on F_b are not polarized and do not exert any torque. However, those which are reflected by F_a get polarized and do exert a torque on F_b. Since they are polarized after a reflection instead of a

transmission as in the previous case, their polarization, hence the torque, is of opposite sign. Thus this torque tends to *anti-align* the magnetization of F_b with respect to the one of F_a. By using a strong enough current, one should be able to switch back and forth the direction of the magnetization of F_b, in the absence of any external magnetic field.

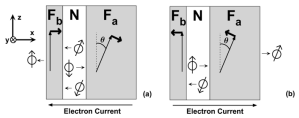

FIG. 4: Magnetic threelayer. One layer (F_a) is used as a polarizer while the torque can switch the direction of the magnetization of other one (F_b).

B. Transport in a magnetic domain wall. Spin injection and Larmor precession.

The previous analysis can also be done in the case of a magnetic domain wall in a ferromagnetic wire. It is however, slightly more subtle since now, we need to keep track of the magnetization direction which is no longer constant in space and we will stick here to more qualitative arguments. Let λ_w be the length of the domain wall (i.e. the length over which the direction of the magnetization is fully reversed). If λ_w is large enough, the change in the magnetization direction is very slow, and the electrons will have time to adapt there spin *adiabatically* to the local direction of the magnetization. The question that arises naturally is how does this adaptation take place. It could, in principle, be due to all kind of relaxation processes (like spin-orbit, coupling to phonons...). There is however, a faster mechanism, namely Larmor precession. When an electron enters the wall (Fig.5a), the magnetization direction starts to rotate and an angle α starts to build between it and the electron's spin (Fig.5b). Hence, the spin start to precess around the magnetization (Fig.5c). This precession take place on a length scale λ_L (Larmor precession length, roughly the inverse of the difference between the Fermi momentum of the majority and minority spins). After $\lambda_L/2$, the spin has made half a roundtrip around the magnetization (Fig.5d) and the angle α is now decreasing. Hence the mistracking of the electron's spin is roughly $\alpha \approx \pi\lambda_L/\lambda_w$. Let ρ_M (ρ_m) be the resistance of the majority (minority) channel for an infinite wall ($\alpha = 0$). The resistance ρ in that case is obtained by adding the two resistances in parallel, $\rho = \rho_M\rho_m/(\rho_m + \rho_M)$. When $\alpha \neq 0$ however, the domain wall causes a slight mixing of the two channels (i.e they are partly in parallel) which causes an increase $\Delta\rho \approx \alpha(\rho_M - \rho_m)^2/(\rho_M + \rho_m)$ of resistance. In addition, it is immediate to realize that a spin going adiabatically through a domain wall is fully flipped in the process, giving $\hbar$ of angular momentum to the wall magnetization. It results that a global pressure is exerted on the wall, pushing it in the direction of the electronic flow.

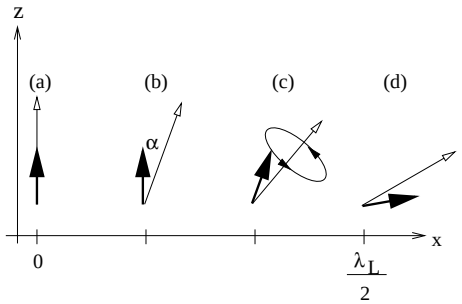

FIG. 5: Competition between the rotation of the magnetization and Larmor precession in a domain wall. The thick arrow stands for the electron spin while the thin one stands for the magnetization.

III. CONCLUSION AND BIBLIOGRAPHY

The reader interested in mesoscopic physics in general might refer to [1] and to [2, 3] for the application of Scattering theory to transport. The spin torque was first predicted by Slonczewski in [4] and the first experiments done in [5].

The scattering formulation of the spin torque is done in [6]. Domain wall physics can be looked at in [7].

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